Web Appendix (Not intended for publication) for: The Life-cycle Growth of Plants: The Role of Productivity, Demand and Wedges.

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February 10, 2023

A Price Indices

A.1 CUPI Price Index

Our baseline results use Redding and Weinstein's (2020) CUPI price indices at the plant level as deflators. Here, we follow Redding and Weinstein (2020) to derive the CUPI index in the context of our model. The change in prices from one period to the next in our model is:

$$\frac{P_{ft}}{P_{ft-1}} = \left(\frac{\sum_{\Omega_t^f} d_{fjt}^{\sigma_w} p_{fjt}^{1-\sigma_w}}{\sum_{\Omega_{t-1}^f} d_{fjt-1}^{\sigma_w} p_{fjt-1}^{1-\sigma_w}} \right)^{\frac{1}{1-\sigma_w}} \tag{1}$$

Defining as $\Omega_{t,t-1}^f$ the set of goods that is common to both periods, and multiplying both the numerator and the denominator by
$\left(\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma_w} p_{fjt-1}^{1-\sigma_w} * \sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma_w} p_{fjt}^{1-\sigma_w} \right)^{\frac{1}{1-\sigma_w}}$ we obtain:

$$\begin{aligned}\frac{P_{ft}}{P_{ft-1}} &= \left(\frac{\sum_{\Omega_t^f} d_{fjt}^{\sigma_w} p_{fjt}^{1-\sigma_w}}{\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma_w} p_{fjt}^{1-\sigma_w}} \frac{\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma_w} p_{fjt-1}^{1-\sigma_w}}{\sum_{\Omega_{t-1}^f} d_{fjt-1}^{\sigma_w} p_{fjt-1}^{1-\sigma_w}} \frac{\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma_w} p_{fjt}^{1-\sigma_w}}{\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma_w} p_{fjt-1}^{1-\sigma_w}} \right)^{\frac{1}{1-\sigma_w}} && (2)\\ &= \frac{\lambda_{ft-1,\Omega_{t,t-1}^f}}{\lambda_{ft,\Omega_{t,t-1}^f}} \left(\frac{\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma_w} p_{fjt}^{1-\sigma_w}}{\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma_w} p_{fjt-1}^{1-\sigma_w}} \right)^{\frac{1}{1-\sigma_w}} && (3)\end{aligned}$$

where $\lambda_{ft-1,\Omega_{t,t-1}^f} = \left(\frac{\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma_w} p_{fjt-1}^{1-\sigma_w}}{\sum_{\Omega_{t-1}^f} d_{fjt-1}^{\sigma_w} p_{fjt-1}^{1-\sigma_w}} \right)^{\frac{1}{1-\sigma_w}}$ and $\lambda_{ft,\Omega_{t,t-1}^f} = \left(\frac{\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma_w} p_{fjt}^{1-\sigma_w}}{\sum_{\Omega_t^f} d_{fjt}^{\sigma_w} p_{fjt}^{1-\sigma_w}} \right)^{\frac{1}{1-\sigma_w}}$.

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Furthermore, since

$$s_{fjt} = \frac{p_{fjt} q_{fjt}}{R_{ft}} = \frac{p_{fjt}^{1-\sigma_w} \left(d_{fjt}^{\sigma_w} \right)}{P_{ft}^{1-\sigma_w}} \tag{4}$$

we have that:

$$\lambda_{ft-1,\Omega_{t,t-1}^f} = \left(\sum_{\Omega_{t,t-1}^f} \frac{d_{fjt-1}^{\sigma_w} p_{fjt-1}^{1-\sigma_w}}{\sum_{\Omega_{t-1}^f} d_{fjt-1}^{\sigma_w} p_{fjt-1}^{1-\sigma_w}} \right)^{\frac{1}{1-\sigma_w}} = \left(\sum_{\Omega_{t,t-1}^f} s_{fjt-1} \right)^{\frac{1}{1-\sigma_w}}$$

That is, $\left(\lambda_{ft-1,\Omega_{t,t-1}^f}\right)^{1-\sigma_w}$ is the share of period $t-1$ expenditures devoted to goods that are common to both periods. Similarly, $\left(\lambda_{ft,\Omega_{t,t-1}^f}\right)^{1-\sigma_w}$ is the share of period t expenditure devoted to goods common to both periods.

With this, the change in prices between the two periods (equation (1)) can be written:

$$\frac{P_{ft}}{P_{ft-1}} = \left(\frac{\sum_{\Omega_{t,t-1}^f} s_{fjt}}{\sum_{\Omega_{t,t-1}^f} s_{fjt-1}} \right)^{\frac{1}{\sigma_w - 1}} \frac{P_{ft}^*}{P_{ft-1,\Omega_{t,t-1}^f}^*} = \left(\lambda_{ft}^{QF} \right)^{\frac{1}{\sigma_w-1}} \frac{P_{ft}^*}{P_{ft-1,\Omega_{t,t-1}^f}^*} \tag{5}$$

where $P_{ft}^* = \left(\sum_{\Omega_{t,t-1}^f} d_{fjt}^{\sigma_w} p_{fjt}^{1-\sigma_w} \right)^{\frac{1}{1-\sigma_w}}$ is a period t price index for the basket of goods common to t and $t-1$ for firm f, and $P_{ft-1,\Omega_{t,t-1}^f}^* = \left(\sum_{\Omega_{t,t-1}^f} d_{fjt-1}^{\sigma_w} p_{fjt-1}^{1-\sigma_w} \right)^{\frac{1}{1-\sigma_w}}$ is a period $t-1$ price index for that same basket. Term $\frac{\sum_{\Omega_{t,t-1}^f} s_{fjt}}{\sum_{\Omega_{t,t-1}^f} s_{fjt-1}} = \lambda_{ft}^{QF}$ is the Feenstra (1994) adjustment for changing varieties, expressed in terms of observables.

Moreover, the Marshallian demands, given by $q_{fjt} = d_{ft}^{\sigma_w} d_{fjt}^{\sigma_w} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma_w} \left(\frac{p_{fjt}}{P_{ft}}\right)^{-\sigma_w} \frac{E_t}{P_t}$, imply

$$s_{fjt}^* = \frac{d_{ft}^{\sigma_w} d_{fjt}^{\sigma_w} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma_w} \frac{p_{fjt}^{1-\sigma_w}}{P_{ft}^{-\sigma_w}} \frac{E_t}{P_t}}{\sum_{\Omega_{t,t-1}^f} d_{ft}^{\sigma_w} d_{fjt}^{\sigma_w} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma_w} \frac{p_{fjt}^{1-\sigma_w}}{P_{ft}^{-\sigma_w}} \frac{E_t}{P_t}} = \frac{d_{fjt}^{\sigma_w} p_{fjt}^{1-\sigma_w}}{\left(P_{ft}^*\right)^{1-\sigma_w}}$$

and

$$s_{fjt-1,\Omega_{t,t-1}^f}^* = \frac{d_{ft-1}^{\sigma_w} d_{fjt-1}^{\sigma_w} \left(\frac{P_{ft-1}}{P_{t-1}}\right)^{-\sigma_w} \frac{p_{fjt-1}^{1-\sigma_w}}{P_{ft-1}^{-\sigma_w}} \frac{E_{t-1}}{P_{t-1}}}{\sum_{\Omega_{t,t-1}^f} d_{ft-1}^{\sigma_w} d_{fjt-1}^{\sigma_w} \left(\frac{P_{ft-1}}{P_{t-1}}\right)^{-\sigma_w} \frac{p_{fjt-1}^{1-\sigma_w}}{P_{ft-1}^{-\sigma_w}} \frac{E_{t-1}}{P_{t-1}}} = \frac{d_{fjt-1}^{\sigma_w} p_{fjt-1}^{1-\sigma_w}}{\left(P_{ft-1,\Omega_{t,t-1}^f}^*\right)^{1-\sigma_w}}$$

Dividing s_{fjt}^* by $s_{fjt-1,\Omega_{t,t-1}^f}^*$ and rearranging, we obtain

$$\left(\frac{p_{fjt}}{p_{fjt-1}}\right) = \left(\frac{P_{ft}^*}{P_{ft-1,\Omega_{t,t-1}^f}^*}\right) \left(\frac{s_{fjt}^*}{s_{fjt-1,\Omega_{t,t-1}^f}^*}\right)^{\frac{1}{1-\sigma_w}} \left(\frac{d_{fjt}}{d_{fjt-1}}\right)^{-\frac{\sigma_w}{1-\sigma_w}}$$

Given this, for plant-product weights $\omega_{ft} = \frac{1}{\left\|\Omega^f_{t,t-1}\right\|}$ such that $\sum_{\Omega^f_{t,t-1}} \omega_{ft,t-1} = 1$ we can write,

$$\begin{aligned} &\sum_{\Omega^f_{t,t-1}} \ln\left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} \\ = \; &\ln\left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right) + \frac{1}{(1-\sigma_w)} \sum_{\Omega^f_{t,t-1}} \ln\left(\frac{s^*_{fjt}}{s^*_{fjt-1,\Omega^f_{t,t-1}}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} \\ &+ \frac{\sigma_w}{\sigma_w - 1} \sum_{\Omega^f_{t,t-1}} \ln\left(\frac{d_{fjt}}{d_{fjt-1}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} \end{aligned}$$

where the first right-hand-side term takes into account that $\sum_{\Omega^f_{t,t-1}} s^*_{fjt,\Omega^f_{t,t-1}} = 1$. Shocks d_{fjt} have been defined relative to plant appeal, d_{ft}, such that $\prod_{\Omega^f_{t,t-1}} d_{fjt}^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} = 1$, with the implication that

$\sum_{\Omega^f_{t,t-1}} \ln\left(\frac{d_{fjt}}{d_{fjt-1}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} = 0$. Notice that this normalization still allows for a distribution of product appeal that varies over time. [1]

The consecutively common good price index growth $\left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right)$ therefore corresponds to

$$\begin{aligned} \ln\left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right) &= \sum_{\Omega^f_{t,t-1}} \ln\left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} - \frac{1}{(1-\sigma_w)} \sum_{\Omega^f_{t,t-1}} \ln\left(\frac{s^*_{fjt}}{s^*_{fjt-1,\Omega^f_{t,t-1}}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} \\ &= \sum_{\Omega^f_{t,t-1}} \ln\left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} - \frac{1}{(1-\sigma_w)} \ln \lambda^{QRW}_{ft} \end{aligned}$$

The term $\ln \lambda^{QRW}_{ft} = \sum_{\Omega^f_{t,t-1}} \ln\left(\frac{s^*_{fjt}}{s^*_{fjt-1,\Omega^f_{t,t-1}}}\right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}}$ adjusts for changes in appeal for continuing products, addressing the consumer valuation bias. Plugging into equation (5), we

[1] This is by contrast to empirical price indices that weight across products with variable weights $\omega_{fjt} \neq \omega_{ft}$, such as the commonly used Sato-Vartia approach (Sato, 1976; Vartia, 1976; Feenstra, 1994). Under such variable weights the assumption $\sum_{\Omega^f_{t,t-1}} \ln\left(\frac{d_{fjt}}{d_{fjt-1}}\right)^{\omega_{fjt}} = 0$ does not hold. The fact that traditional approaches using variable weights ignore this term leads to what Redding and Weinstein (2020) have called the "consumer valuation bias" the traditional empirical approaches to economically motivated price indices.

obtain

$$\ln \frac{P_{ft}}{P_{ft-1}} = \sum_{\Omega^f_{t,t-1}} \ln \left(\frac{p_{fjt}}{p_{fjt-1}} \right)^{\frac{1}{\left\|\Omega^f_{t,t-1}\right\|}} - \frac{1}{(1-\sigma_w)} \left(\ln \lambda^{QRW}_{ft} + \ln \lambda^{QF}_{ft} \right) \tag{6}$$

We similarly obtain a measure of materials by deflating material expenditure by plant-level price indices for materials, pm_{ft}, using information on individual prices and quantities of material inputs. We construct pm_{ft} using an analogous approach to that used to construct output prices. The underlying assumption is that M_{ft}, the index of materials quantities used, is a CES aggregate of individual inputs. As is the case with output prices, until we have an estimate of the elasticity of substitution, we can only build a consecutively-common-basket price index $\overline{pm^*_{ft}}$ for plant f, and carry an adjustment factor $\Lambda^M_{ft} = \Lambda^{MRW}_{ft} \Lambda^{MF}_{ft}$ for which we later adjust prices. In particular, we deflate materials expenditures to obtain $M^*_{ft} = \frac{\text{materials expenditure}_{ft}}{pm_{fB}\overline{pm^*_{ft}}} = M_{ft} * \left(\Lambda^M_{ft}\right)^{\frac{1}{\sigma_w - 1}}$. Once we have obtained an estimate of the elasticity of substitution we calculate $pm_{ft} = pm_{fB} * \overline{pm^*_{ft}} * \left(\Lambda^M_{ft}\right)^{\frac{1}{\sigma_w - 1}}$, which is one of the attributes on the cost side in our growth decomposition. We use this price index as deflator for materials expenditure to obtain our $TFPQ$ measure. We use for inputs the same elasticity of substitution estimated for outputs. We recognize that using the same elasticity for inputs and outputs is a strong assumption, but find that it does no affect our results in an important way.

A.2 Initializing a Plant's CUPI Price Index

A plant's price index is constructed as

$$P_{ft} = P_{fB} * \overline{P^*_{ft}} * \left(\Lambda^Q_{ft}\right)^{\frac{1}{\sigma_w - 1}}$$

The initial level P_{fB}, where B is the base year for plant f, is constructed as: $P_{fB} = P_{base,B} \prod_{\Omega^f_B} \left(\frac{p_{fjB}}{\overline{p_{jB}}} \right)^{s_{fjB}}$, where $\overline{p_{jB}}$ is the geometric average of the price of product j in year B across plants, year B is the first year in which plant f is present in the survey, and $P_{base,B}$ is an overall base. We use 1982 as the base year, so $P_{base,1982} = 1$. For plants with $B \neq 1982$, $P_{base,B}$ is set equal to the geometric mean of the price index across plants that we observe prior to year B. Notice that our approach takes advantage of cross sectional variability across plants for any given product or input j. In the plant's base year B, $\left(\frac{P_{fjB}}{\overline{P_{jB}}} \right) = 1$ for the average producer of product j. For other plants, it will capture dispersion in price levels around that average.[2]

[2] We deal with excessive noise from partial-year reporting and other sources by eliminating outliers. In particular, in any given year we consider only products that represent at least 2% of sales of the respective plant. Shares are re-calculated accordingly for this restricted basket. We also winsorize the 2% tails at each step of the process of building price indices. In particular, we winsorize $\frac{\sum_{\Omega^f_{l,l-1}} s_{fjl}}{\sum_{\Omega^f_{l,l-1}} s_{fjl-1}}$; $\prod_{\Omega^f_{t,t-1}} \left(\frac{s^*_{fjt}}{s^*_{fjt-1,\Omega^f_{t,t-1}}} \right)^{\frac{1}{\left\|\Omega_{t,t-1}\right\|}}$;

B Firm's Problem

Firm chooses X_t to solve:

$$\underset{\{X_{ft}\}}{Max} \quad \pi_{ft} = (1-\tau_{ft}) R_{ft} - C_{ft}X_{ft} = (1-\tau_{ft}) D_{ft} A_{ft}^{1-\frac{1}{\sigma}} X_{ft}^{\gamma\left(1-\frac{1}{\sigma}\right)} - C_{ft}X_{ft}$$

where $R_{ft} = P_{ft}Q_{ft} = D_{ft}Q_{ft}^{1-\frac{1}{\sigma}} = d_{ft}E_t^{\frac{1}{\sigma}}P_t^{1-\frac{1}{\sigma}}Q_{ft}^{1-\frac{1}{\sigma}}$ and $Q_{ft} = A_{ft}X_{ft}^{\gamma}$. Optimal input demand is

$$X_{ft} = \left(\frac{(1-\tau_{ft}) D_{ft} A_{ft}^{1-\frac{1}{\sigma}}\gamma}{\mu_{ft}C_{ft}}\right)^{\frac{1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}} \tag{7}$$

Proof. If the firm has market power, then $\frac{\partial P_t}{\partial X_{ft}} \neq 0$. The first order condition for the firm is then given by

$$\begin{aligned}
(1-\tau_{ft})\left(1-\frac{1}{\sigma}\right)\left(\frac{R_{ft}}{Q_{ft}} + \frac{R_{ft}}{P_{ft}}\frac{\partial P_t}{\partial Q_{ft}}\right)\frac{\partial Q_{ft}}{\partial X_{ft}} &= C_{ft} \\
(1-\tau_{ft})\left(1-\frac{1}{\sigma}\right)\frac{R_{ft}}{Q_{ft}}\left(1 + \frac{Q_{ft}}{P_{ft}}\frac{\partial P_t}{\partial Q_{ft}}\right)\frac{\partial Q_{ft}}{\partial X_{ft}} &= C_{ft} \\
(1-\tau_{ft})\left(\frac{\sigma-1}{\sigma}\right) D_{ft}Q_{ft}^{-\frac{1}{\sigma}}(1-s_{ft})\frac{\partial Q_{ft}}{\partial X_{ft}} &= C_{ft} \\
\frac{(1-\tau_{ft})}{\mu_{ft}} D_{ft}Q_{ft}^{-\frac{1}{\sigma}}\left(\gamma A_{ft}X_{ft}^{\gamma-1}\right) &= C_{ft} \qquad (8)\\
\frac{(1-\tau_{ft}) D_{ft} A_{ft}^{1-\frac{1}{\sigma}}\gamma}{\mu_{ft}C_{ft}} &= X_{ft}^{1-\gamma\left(1-\frac{1}{\sigma}\right)} \qquad (9)
\end{aligned}$$

Where the third line uses Sheppard's lemma $\left(-\frac{\partial P_t}{\partial Q_{ft}}\frac{Q_{ft}}{P_t} = s_{ft}\right)$, and the fourth line uses $\mu^{-1} = 1 - \left(\frac{1}{\sigma} + \left(\frac{\sigma-1}{\sigma}\right)s_{ft}\right) = \frac{\sigma-1-(\sigma-1)s_{ft}}{\sigma} = \frac{(\sigma-1)\left(1-s_{ft}\right)}{\sigma}$ (see Appendix D). Therefore $X_{ft} = \left(\frac{\left(1-\tau_{ft}\right)D_{ft}A_{ft}^{1-\frac{1}{\sigma}}\gamma}{\mu_{ft}C_{ft}}\right)^{\frac{1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}}$. ■

Suppose $X_{ft} = K_{ft}^{\frac{\alpha}{\gamma}}L_{ft}^{\frac{\beta}{\gamma}}M_{ft}^{\frac{\phi}{\gamma}}$ where K, L and M are, respectively, capital, labor, and material inputs, and $\gamma = \alpha+\beta+\phi$. Consequently, C_{ft} is itself a Cobb Douglas aggregate of factor prices: $C_{ft} = r_{ft}^{\frac{\alpha}{\gamma}}w_{ft}^{\frac{\beta}{\gamma}}pm_{ft}^{\frac{\phi}{\gamma}}$. We note that we do not have information on the rental price of capital, which then goes into wedges (see below). Consequently,

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$\frac{p_{fjt}}{p_{fjt-1}}$; $\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}$; $\frac{P_{ft}}{P_{ft-1}}$.

We also winsorize adjustment factors at the 5% level. Extreme changes in the baskets of goods, where common (t,$t-1$) products represent a negligible share of revenue in either t or $t-1$ imply extreme values for $\ln \Lambda^Q_{ft}$. These extreme changes may partly reflect measurement error in an enviroment where baskets of goods are auto-reported into relatively wide product components.

$$X_{ft} = d_{ft}^{\kappa_1} a_{ft}^{\kappa_2} pm_{ft}^{-\frac{\phi}{\gamma}\kappa_1} w_{ft}^{-\frac{\beta}{\gamma}\kappa_1} \mu_{ft}^{-\kappa_1} \kappa_t \widehat{\kappa}_{ft} \quad (10)$$

$$Q_{ft} = d_{ft}^{\gamma\kappa_1} a_{ft}^{\kappa_1} pm_{ft}^{-\phi\kappa_1} w_{ft}^{-\beta\kappa_1} \mu_{ft}^{-\gamma\kappa_1} \chi_t \chi_{ft} \quad (11)$$

where $\kappa_1 = \frac{1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}$; $\kappa_2 = \left(1-\frac{1}{\sigma}\right)\kappa_1$; $\chi_t = D_t^{\gamma\kappa_1} A_t^{\kappa_1} C_t^{-\gamma\kappa_1} \mu_t)^{-\gamma\kappa_1}$ captures aggregate growth between birth and age t, and $\chi_{ft} = (1-\tau_{ft})^{\gamma\kappa_1} r'^{\,-\alpha\kappa_1}_{ft}$ captures residual variation from wedges, and the unobserved user cost of capital (adjusted for factor-biased distortions), r'_{ft}. Additionally, $\widehat{\kappa}_{ft} = \chi_{ft}^{\frac{1}{\gamma}}$; $\kappa_t = \chi_t^{\frac{1}{\gamma}} A_t^{-1}$. We have used the fact that $1 + \gamma\kappa_1\left(1-\frac{1}{\sigma}\right) = \kappa_1$.

Moreover, since $R_{ft} = D_{ft} Q_{ft}^{1-\frac{1}{\sigma}}$ and $1 + \gamma\kappa_1\left(1-\frac{1}{\sigma}\right) = \kappa_1$ then

$$R_{ft} = d_{ft}^{\kappa_1} a_{ft}^{\kappa_2} pm_{ft}^{-\phi\kappa_2} w_{ft}^{-\beta\kappa_2} \mu_{ft}^{-\gamma\kappa_2} \left(\widehat{\chi}_t \chi_{ft}\right)^{1-\frac{1}{\sigma}}$$

Notice also that 8 can be re-written as:

$$\begin{aligned} \frac{(1-\tau_{ft})}{\mu_{ft}} D_{ft} Q_{ft}^{-\frac{1}{\sigma}} \theta_{ft}^{v} &= C_{ft} \\ \frac{(1-\tau_{ft})}{\mu_{ft}} P_{ft} \theta_{ft}^{v} &= \frac{C_{ft} X_{ft}}{Q_{ft}} \\ \frac{\theta_{ft}^{v}}{\frac{C_{ft} X_{ft}}{P_{ft} Q_{ft}}} &= \frac{\mu_{ft}}{(1-\tau_{ft})} \end{aligned}$$

where θ_{ft}^{v} is the output elasticity of a variable factor V and we have used $P_{ft} = D_{ft} Q_{ft}^{-\frac{1}{\sigma}}$. The expression $\frac{\theta_{ft}^{v}}{\frac{C_{ft} X_{ft}}{P_{ft} Q_{ft}}}$ is markup "a-la-De Loecker", which is here shown to equal a ratio between the model's markup, μ_{ft}, and the revenue wedge $(1-\tau_{ft})$.

Empirically, we estimate $\widehat{\mu}_{ft}^{DL} = \frac{\phi}{\frac{pm_{ft} M_{ft}}{R_{ft}}}$, because M is the only variable factor in our estimations. As discussed in the main text, using this expression for the markup μ_{ft}^{DL} yields a discrepancy between the ratio of estimated output elasticity to the cost share of revenue and the ratio of the model's markup and the revenue wedge. This discrepancy can be accounted for by a factor-specific wedge yielding:

$$\widehat{\mu}_{ft}^{DL} = \frac{\phi}{\frac{C_{ft} X_{ft}}{P_{ft} Q_{ft}}} = \frac{\mu_{ft}(1-\tau_{ft}^{v})}{(1-\tau_{ft})}$$

The factor-specific wedge is already implicitly incorporated in the sales wedge as the latter is a composite wedge measure that captures any source of discrepancy between actual and sales implied by the static model based on model parameters and attributes. Factor-specific wedges will have an impact on scale but also will impact factor mix. This implies that there is an additional potential impact of a factor-specific wedge on first-order conditions for individual inputs. This factor-specific wedge may have a variety of sources, including factor-specific frictions and wedges, measurement, and specification issues. The latter includes, for example, differences in the actual vs. estimated factor elasticity for the variable factor. If we use equation (B) and the type of structural decomposition presented in appendix G, we find that about 65% of measured $\widehat{\mu}_{ft}^{DL}$ is accounted for by the factor-specific wedge.

C Supplementary Results

Production function coefficients by sector are shown in Table C.1, while Table C.2 describes the sector classification.

Table C.1: Factor and demand elasticities by sector

Sector	β	α	ϕ	σ_w	σ	σ_w/σ	γ	$\gamma(1-1/\sigma)$
311	0.26	0.11	0.69	3.29	1.82	1.81	1.05	0.47
313	0.26	0.07	0.64	4.14	2.31	1.80	0.98	0.55
321	0.29	0.21	0.49	3.72	2.09	1.78	0.98	0.51
322	0.15	0.10	0.67	4.65	2.57	1.81	0.93	0.57
323	0.24	0.15	0.56	3.25	1.81	1.80	0.94	0.42
324	0.21	0.10	0.66	4.24	2.40	1.77	0.97	0.57
331	0.27	0.13	0.57	3.20	1.74	1.84	0.97	0.41
332	0.29	0.06	0.64	2.90	1.59	1.82	0.98	0.37
341	0.40	0.10	0.58	2.15	1.20	1.79	1.08	0.18
342	0.49	0.14	0.38	2.68	1.42	1.88	1.01	0.30
351	0.43	0.27	0.40	4.98	2.75	1.81	1.09	0.69
352	0.40	0.20	0.49	3.41	1.86	1.83	1.09	0.51
355	0.48	0.03	0.52	4.33	2.38	1.82	1.03	0.60
356	0.38	0.13	0.56	2.52	1.39	1.81	1.07	0.30
362	0.65	0.28	0.23	3.02	1.68	1.80	1.16	0.47
369	0.57	0.20	0.34	4.49	2.54	1.77	1.11	0.68
371	0.51	0.20	0.49	3.28	1.74	1.88	1.20	0.51
381	0.30	0.12	0.54	2.67	1.48	1.80	0.97	0.31
382	0.45	0.07	0.50	3.27	1.82	1.80	1.02	0.46
383	0.29	0.14	0.56	3.46	1.91	1.81	1.00	0.47
384	0.41	0.13	0.53	4.63	2.58	1.79	1.06	0.65
385	0.38	0.19	0.39	3.77	2.07	1.83	0.96	0.49
390	0.32	0.15	0.53	3.24	1.78	1.82	1.00	0.44
Average	0.37	0.14	0.52	3.53	1.95	1.81	1.03	0.48

Table C.2: Sector classifications (3 digit ISIC)

Sector	Description	Observations
311	Food manufacturing (311 and 312).	31,524
313	Beverage industries (313) and Tobacco industries (314).	3,025
321	Manufacture of textiles.	7,619
322	Manufacture of wearing apparel, except footwear.	18,542
323	Manufacture of leather and products of leather, leather substitutes and fur, except footwear and wearing apparel.	2,678
324	Manufacture of footwear, except vulcanized or moulded rubber or plastic footwear.	6,516
331	Manufacture of wood and wood products, except furniture.	4,190
332	Manufacture of furniture and fixtures, except primarily of metal.	8,213
341	Manufacture of paper and paper products.	4,190
342	Printing, publishing and allied industries.	8,576
351	Manufacture of industrial chemicals.	3,276
352	Manufacture of other chemical products (352); Petroleum refineries (353); Manufacture of miscellaneous products of petroleum and coal (354).	10,378
355	Manufacture of rubber products.	1,929
356	Manufacture of plastic products not elsewhere classified.	10,228
362	Manufacture of pottery, china and earthenware (361) and Manufacture of glass and glass products (362).	2,179
369	Manufacture of structural clay products.	5,900
371	Basic metal industries (371 and 372).	2,471
381	Manufacture of cutlery, band tools and general hardware.	13,006
382	Manufacture of machinery except electrical.	8,862
383	Manufacture of electrical machinery, apparatus, appliances and supplies.	5,406
384	Manufacture of transport equipment.	4,125
385	Manufacture of professional and scientific, and measuring and controlling equipment not elsewhere classified, and of photographic and optical goods.	1,350
390	Other manufacturing industries.	4,265

Note: Descriptive statistics are restricted to a sample of plants observations which have information on all measured plant attributes.

Table C.3: Sector classifications for first 15 sectors at 3 digit CPC

Sector	Description	Observations
211	Meat and meat products	3,060
212	Prepared and preserved fish	290
213	Prepared and preserved vegetables	290
214	Fruit juices and vegetable juices	181
215	Prepared and preserved fruit and nuts	842
216	Animal and vegetable oils and fats (216); Cotton linters (217); Oil-cake and other residues resulting from the extraction of vegetable fats or oils; flours and meals of oil seeds or oleaginous fruits, except those of mustard; vegetable waxes, except triglycerides; degras; residues resulting from the treatment of fatty substances or animal or vegetable waxes (218)	1,209
221	Processed liquid milk and cream	458
229	Other dairy products	2,369
231	Grain mill products	4,735
232	Starches and starch products; sugars and sugar syrups n.e.c	225
233	Preparations used in animal feeding	1,529
234	Bakery products	10,795
235	Sugar	633
236	Cocoa, chocolate and sugar confectionery	1,144
237	Pasta, macaroni, noodles, couscous and similar farinaceous products	735

Note: Number of observations are restricted to a sample of plants observations which have information on all measured plant attributes.

Table C.4: Distribution of CPC 3-digit group sizes

	Min	P25	P50	P75	Max	Average
Observations in group	38	375	728	1,545	20,867	1,517
Observations in group-year	2.7	12.1	23.5	49.8	673.1	48.98

Note: Number of observations are restricted to a sample of plants observations which have information on all measured plant attributes.

Table C.5: Distribution of Largest Plants' Markup Relative to Sector*Year Average

	Largest	Second	Third
Average	2.79	1.37	1.23
10	1.07	1.04	1.03
25	1.24	1.14	1.08
50	1.64	1.30	1.18
75	2.39	1.54	1.35
90	3.68	1.74	1.49
95	6.38	1.98	1.60
99	29.38	2.40	1.77
Max	46.36	4.59	2.05
N	713	713	713

Note: There are 23 sectors in 31 years ($23 \times 31 = 713$) which gives one observation per sector*year. Distributions are restricted to a sample of plants observations which have information on all measured plant attributes.

D Markups

Under Cournot competition, the firm's (potentially variable) markup after the distortion, $\mu_{ft} = \frac{P_{ft}}{mc_{ft}\left(1-\tau_{ft}\right)^{-1}}$, is given by:

$$\mu_{ft} = \frac{1}{1 - \left(\frac{1}{\sigma} - \left(\frac{\sigma-1}{\sigma}\right) s_{ft}\right)} = \frac{\sigma}{(\sigma-1)\left(1-s_{ft}\right)} \tag{12}$$

Proof. $\underset{Q_{ft}}{Max}\ (1-\tau_{ft}) P_{ft} Q_{ft} - CT$ leads to first orden condition $\left(P_{ft} + Q_{ft}\frac{dP_{ft}}{dQ_{ft}}\right) = \frac{mc_{ft}}{\left(1-\tau_{ft}\right)}$. Dividing by P_{ft} we obtain $\frac{1}{\mu_{ft}} = 1 + \frac{Q_{ft}}{P_{ft}}\frac{dP_{ft}}{dQ_{ft}} = 1 - \epsilon^{-1}$ (where we have denoted $\epsilon_{ft} \equiv -\frac{Q_{ft}}{P_{ft}}\frac{dP_{ft}}{dQ_{ft}}$), so that

$$\mu_{ft} = \left(\frac{\epsilon_{ft}}{\epsilon_{ft}-1}\right) \tag{13}$$

In turn, under $Q_{ft} = d_{ft}^{\sigma} P_{ft}^{-\sigma} \frac{E_t}{P_t^{1-\sigma}}$ and its implication that $P_{ft} = d_{ft} Q_{ft}^{-\frac{1}{\sigma}} \left(\frac{E_t}{P_t^{1-\sigma}}\right)^{\frac{1}{\sigma}} = d_{ft} Q_{ft}^{-\frac{1}{\sigma}} \left(\frac{Q_t}{P_t^{-\sigma}}\right)^{\frac{1}{\sigma}}$ and allowing for market power so that $\frac{dP_t}{dQ_{ft}} \neq 0$, the inverse of the demand elasticity as perceived by the firm ($\epsilon_{ft}^{-1} \equiv -\frac{dP_{ft}}{dQ_{ft}}\frac{Q_{ft}}{P_{ft}}$) is:

$$\begin{aligned}
\epsilon_{ft}^{-1} &= -\left(\frac{\partial P_{ft}}{\partial Q_{ft}} + \frac{\partial P_{ft}}{\partial P_t}\frac{\partial P_t}{\partial Q_{ft}}\right)\frac{Q_{ft}}{P_{ft}} \qquad (14) \\
&= -\left(-\frac{1}{\sigma}\frac{P_{ft}}{Q_{ft}} + \left(\frac{\sigma-1}{\sigma}\right)\frac{P_{ft}}{P_t}\frac{\partial P_t}{\partial Q_{ft}}\right)\frac{Q_{ft}}{P_{ft}} \\
&= \left(\frac{1}{\sigma} - \left(\frac{\sigma-1}{\sigma}\right)\frac{\partial P_t}{\partial Q_{ft}}\frac{Q_{ft}}{P_t}\right) \\
&= \left(\frac{1}{\sigma} + \left(\frac{\sigma-1}{\sigma}\right)s_{ft}\right) \qquad (15)
\end{aligned}$$

where the last line uses Sheppard's lemma: $-\frac{\partial P_t}{\partial Q_{ft}}\frac{Q_{ft}}{P_t} = s_{ft}$.

Equations (13) and (15) together imply $\mu_{ft}^{-1} = 1-\epsilon_{ft}^{-1} = 1-\left(\frac{1}{\sigma} + \left(\frac{\sigma-1}{\sigma}\right)s_{ft}\right) = \left(\frac{\sigma-1}{\sigma} - \left(\frac{\sigma-1}{\sigma}\right)s_{ft}\right)$, so that

$$\begin{aligned}
\mu_{ft} &= \frac{1}{1-\left(\frac{1}{\sigma} + \left(\frac{\sigma-1}{\sigma}\right)s_{ft}\right)} \\
\mu &= \frac{\sigma}{\sigma-1} \text{ if } s_{ft} = 0
\end{aligned}$$

■

The markup $\mu_{ft} = \frac{\sigma}{(\sigma-1)\left(1-s_{ft}\right)}$ is increasing in the firm's market share. Thus, the markup is itself a function of attributes:

$$\begin{aligned}
s_{ft} &= \frac{P_{ft}Q_{ft}}{E_t} = \frac{D_{ft}Q_{ft}^{1-\frac{1}{\sigma}}}{E_t} = \frac{D_{ft}A_{ft}^{1-\frac{1}{\sigma}}X_{ft}^{\gamma\left(1-\frac{1}{\sigma}\right)}}{E_t} \qquad (16) \\
&= \frac{D_{ft}A_{ft}^{1-\frac{1}{\sigma}}}{E_t}\left(\frac{\gamma\left(1-\tau_{ft}\right)\left(1-\frac{1}{\sigma}\right)D_{ft}A_{ft}^{1-\frac{1}{\sigma}}}{C_{ft}\mu_{ft}\left(\frac{\sigma-1}{\sigma}\right)}\right)^{\frac{\gamma\left(1-\frac{1}{\sigma}\right)}{1-\gamma\left(1-\frac{1}{\sigma}\right)}} \qquad (17)
\end{aligned}$$

so that

$$s_{ft}\left(\frac{\sigma-(\sigma-1)s_{ft}}{\sigma-(\sigma-1)s_{ft}-1}\right)^{\frac{\gamma\left(1-\frac{1}{\sigma}\right)}{1-\gamma\left(1-\frac{1}{\sigma}\right)}} = \frac{D_{ft}^{\frac{1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}}A_{ft}^{\frac{1-\frac{1}{\sigma}}{1-\gamma\left(1-\frac{1}{\sigma}\right)}}}{E_t}\left(\frac{\gamma\left(1-\tau_{ft}\right)\left(1-\frac{1}{\sigma}\right)}{C_{ft}\left(\frac{\sigma-1}{\sigma}\right)}\right)^{\frac{\gamma\left(1-\frac{1}{\sigma}\right)}{1-\gamma\left(1-\frac{1}{\sigma}\right)}}$$

The LHS is increasing in s and the RHS is increasing in D and A, and decreasing in τ and C. Thus, s_{ft} and the markup are increasing in D and A, and decreasing in τ and C.

E Efficiency, Quality/appeal and Endogenous Innovation

Firm choices depend on productivity components such as D_{ft} and A_{ft}. We take them as given when a firm chooses its size, but note that our results should help guide future work, both theoretical and empirical, about the specific drivers of measured productivity. To further understand the nature of $TFPQ$ vs. quality/appeal, and potential mechanisms through which businesses accumulate each of them, we have studied attributes of these attributes, including how they correlate with different types of innovation efforts that can be inferred from reports in the manufacturing survey.

Table E1 presents univariate regressions of D_{ft} and A_{ft} on indicators of efforts to innovate in process, product, or the relationship with clients. Information obtained from the Technological Development and Innovation Survey and the AMS(DANE, 2003-2013*b*, 1982-2013*a*). Both D_{ft} and A_{ft} have been standardized in order to make an appropriate comparison of coefficient magnitudes. We use dummy variables for whether there is an investment spike (defined as a rate of investment in physical assets to initial capital above 25%); whether the plant receives orders online; whether the plant introduced a product that is new to the plant and for which it charges a high-price (above the 75th percentile of the product class); or took an action such that the price of an existing product increased from below the median in its class to above the 75th percentile. The last two indicators are built from the record of individual products that we also use to build price indices. We also regress A_{ft} and D_{ft} (individually) on the (log) value of spending on advertisement and spending on R&D.

Conditioning on sales (which induce a correlation between A_{ft} and D_{ft}) efficiency is positively correlated with investment: A_{ft} is 0.07 standard deviations (s.d.) higher when the plant undergoes an investment spike, which may imply purchasing machinery that is more efficient in production. Interestingly, efforts that suggest product innovation are strongly negatively correlated with efficiency, suggesting that producing more quality may be costly in terms of quantities produced. For instance, conditional on sales, the introduction of a new high price product is related to a 0.21 s.d. decrease in efficiency. Meanwhile, quality/appeal displays positive correlations with all the innovation efforts recorded in Table E1. Conditional on sales, these correlations are particularly large with the indicators for the introduction of a high price product (0.13 s.d.) and for innovations such that an existing product becomes high price (0.10 s.d.).

Table E.1: Correlations From Univariate Regressions Between Observables and TFPQ and Quality/Appeal

	Dependent variable	D (quality/appeal)			TFPQ (technical efficiency)		
		Conditioned on sales, contemporaneous regressor	Not conditioned on sales, contemporaneous regressor	Not conditioned on sales, lagged regressor	Conditioned on sales, contemporaneous regressor	Not conditioned on sales, contemporaneous regressor	Not conditioned on sales, lagged regressor
		(1)	(2)	(3)	(4)	(5)	(6)
Each row corresponds to a different regression	Investment spike (investment rate > 25%)	0.0208***	0.2629***	0.2659***	0.0693***	0.1157***	0.0198***
		(0.0023)	(0.0061)	(0.0065)	(0.0062)	(0.0062)	(0.0066)
	Plant introduced new product of high price (> 75%)	0.1254***	0.2929***	0.2843***	-0.2073***	-0.1748***	-0.1488***
		(0.0062)	(0.0161)	(0.0172)	(0.0162)	(0.0166)	(0.0179)
	Increased price of product from low to high price (< 50% to > 75%)	0.0988***	0.2604***	0.2209***	-0.2415***	-0.2101***	-0.1230***
		(0.0045)	(0.0134)	(0.0145)	(0.0132)	(0.0136)	(0.0148)
	Advertisement spending (logs)	0.0188***	0.3042***	0.3031***	-0.0399***	0.0341***	0.0302***
		(0.0007)	(0.0010)	(0.0011)	(0.0018)	(0.0013)	(0.0014)
	Total investment in R&D (logs)	-0.0004	0.2722***	0.2734***	-0.0247***	0.0360***	0.0360***
		(0.0015)	(0.0024)	(0.0025)	(0.0041)	(0.0030)	(0.0030)
	Internet used for customer support	0.0288***	0.4384***	0.4519***	-0.0480***	0.0247**	0.0229*
		(0.0051)	(0.0118)	(0.0136)	(0.0124)	(0.0122)	(0.0138)
	Sector*Time FE	Yes	Yes	Yes	Yes	Yes	Yes
	Controlling for sales	Yes	No	No	Yes	No	No

Note: D_{ft} and A_{ft} have been standardized in order to compare the coefficients across both outcomes in terms of standard deviations of each variable. Information on the introduction of products and price changes is available since 1983 (the second year in our sample), the information on advertisement spending and investment is available for the entire sample (1982-2012), investment in R&D information is available for the sub-period 2003-2012, and information on internet is available for the sub-period 2008-2012. Results restrict to a sample of plants observations which have information on all measured plant attributes. Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table E2 presents an OLS analysis of the persistence in wedges, and the role of lagged wedges for the evolution of sales, output, $TFPQ$, and demand. Wedges are standardized to facilitate interpretation. Residual wedges exhibit considerable positive persistence but less so than capabilities captured by A_{ft} and D_{ft}. This is consistent with residual wedges in part reflecting non-convex adjustment costs. Such costs generate a wedge that is correlated with attributes and that only persists up to the moment in which the benefit of adjusting overcomes its fixed cost.

As in models of endogenous attributes, contemporaneous attributes and wedges correlate with higher *lagged* wedges (higher implicit lagged subsidies), even after controlling for persistence in attributes, but wedges do not account for much variation in outcomes and attributes. For example, a one standard deviation increase in lagged residual wedges yields a 0.06 increase in $TFPQ$ and a 0.02 increase in demand. These are small effects relative to the standard deviations of $TFPQ$ and demand reported in Table 2 (0.76 and 0.9, respectively).[3] In turn, as hypothesized, the interaction effect between the lagged dependent variable and lagged residual wedges (negatively correlated with lagged attributes, as seen above) is negative. That is, while higher lagged residual wedges are associated with higher outcomes and attributes, they correlate with reduced persistence in outcomes and attributes. But, the interacted effects are also very small.

Results from our decomposition of sales show that, given attributes, high-attributes plants are being implicitly taxed while low-attributes plants are implicitly subsidized (by the envi-

[3] Lagged wedges also exhibit modest correlation with current output and sales.

Table E.2: Residual Wedge and Attributes Persistence

VARIABLES	(1) Residual Wedge (subsidy)	(2) Output	(3) Sales	(4) TFPQ	(5) Demand shock
Lagged Dependent Variable		0.988***	0.991***	0.934***	0.988***
		(0.001)	(0.001)	(0.001)	(0.001)
Lagged residual wedge (subsidy, standarized)	0.760***	0.040***	0.045***	0.059***	0.024***
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
Lagged residual wedge (subsidy, standarized)*Lagged DV		-0.009***	-0.010***	-0.009***	-0.011***
		(0.001)	(0.001)	(0.001)	(0.001)
Observations	145,158	145,158	145,158	145,158	145,158
R-squared	0.560	0.945	0.948	0.803	0.942
Sector*Time FE	Yes	Yes	Yes	Yes	Yes

Note: Results restrict to a sample of plants observations which have information on all measured plant attributes. Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

ronment, not necessarily by the government). Causality in the opposite direction is also likely and supported by the theory: technical efficiency and product-plant appeal, while partly determined by exogenous stochastic dynamics (as in, e.g., Hopenhayn, 2016; Hopenhayn and Rogerson, 1993), partly also result from endogenous investments to improve performance (as in Acemoglu et al., 2018; Aw, Roberts and Xu, 2011). In the latter class of models, firms invest in future attributes (e.g., via R&D expenditure) to the extent that they expect high returns from such investments. High attributes plants should, therefore, invest more in a context with persistence in attributes. Since wedges make future profitability less dependent in attributes, they should reduce the incentive to invest given by high attributes, especially if wedges are negatively correlated with attributes (e.g. Hsieh and Klenow, 2014). Wedges may also have a direct effect on investment if, for instance, the presence of fixed costs of production implies that a subsidy directly increases the chances of surviving to enjoy the returns from R&D. Our results in this section align with these ideas.

F Details for the Joint Estimation of Production and Demand Functions

As in proxy methods for the estimation of the production function, the joint estimation of production and demand is preceded by a first stage that ensures that $TFPQ$ can be proxied by an observable factor, in this case, materials, which is conditionally monotonic in $TFPQ$. The free input M_{ft} is a function of $TFPQ_{ft}$, conditional on quasi-fixed inputs. The FOC for materials is

$$
\begin{aligned}
M_{ft} &= \frac{\phi(1-\tau_{ft})R_{ft}}{pm_{ft}}(1-1/\sigma) \\
&= \frac{\phi(1-\tau_{ft})P_{ft}Q_{ft}}{pm_{ft}}(1-1/\sigma) \\
M_{ft}^{1-\phi} &= \frac{P_{ft}A_{ft}K_{ft}^{\alpha}L_{ft}^{\beta}(1-\tau_{ft})(\phi\frac{\sigma-1}{\sigma})}{pm_{ft}}
\end{aligned}
$$

Within a sector, ϕ and σ display no variability. We thus re-write

$$
\ln M_{ft} = h\left(\ln A_{ft}, \ln K_{ft}, \ln L_{ft}, \ln \frac{\overline{P^{*}_{ft}}P_{fB}}{\overline{PM^{*}_{ft}}PM_{fB}}, \ln \Lambda^{Q}_{ft}, \ln \Lambda^{m}_{ft},\ \ln s_{ft}\right)
$$

We have included s_{ft} since we do not observe τ but know that all firm choices that ultimately feed into s_{ft} are a function of τ (we have measures for all the other variable terms in the material's FOC). In particular, we condition on a flexible polynomial on s_{ft} rather than τ_{ft}. Furthermore, we have used $P_{ft} = \overline{P^{*}_{ft}}P_{fB}\left(\Lambda^{Q}_{ft}\right)^{\frac{1}{\sigma-1}}$ and $pm_{ft} = \overline{PM^{*}_{ft}}PM_{fB}\left(\Lambda^{M}_{ft}\right)^{\frac{1}{\sigma-1}}$. Inverting, we obtain

$$
\ln A_{ft} = h^{-1}\left(\ln M_{ft}, \ln K_{ft}, \ln L_{ft}, \ln \frac{\overline{P^{*}_{ft}}P_{fB}}{\overline{PM^{*}_{ft}}PM_{fB}}, \ln \Lambda^{Q}_{ft}, \ln \Lambda^{m}_{ft}, \ln s_{ft}\right) \equiv h^{-1}\left(\overrightarrow{W}\right) \quad (18)
$$

Incorporating this expression, recognizing that Q_{ft} is subject to measurement error and other shocks not observed by either the econometrician or the firm at the time of making input choices, and denoting by $\widehat{Q_{ft}} = Q_{ft}\varepsilon_{ft}$ measured Q_{ft}, we write:

$$
\widehat{Q_{ft}} = \alpha \ln K_{ft} + \beta \ln L_{ft} + \phi \ln M_{ft} + h^{-1}\left(\overrightarrow{W}\right) + \varepsilon_{ft} \quad (19)
$$

so that

$$
\widehat{Q^{*}_{ft}} = \alpha \ln K_{ft} + \beta \ln L_{ft} + \phi \ln M_{ft} - \frac{1}{\sigma-1}\ln \Lambda^{Q}_{ft} + \frac{\phi}{\sigma-1}\ln \Lambda^{M}_{ft} + h^{-1}\left(\overrightarrow{W}\right) + \varepsilon_{ft} \quad (20)
$$

where ε_{ft} is measurement error, and the "$*$" refers to the fact that we are estimating the transformed $Q^{*}_{ft} = \frac{R_{ft}}{P^{*}_{ft}}$ rather than $Q_{ft} = \frac{R_{ft}}{P_{ft}}$.

In the first stage, we proxy productivity and eliminate measurement error by estimating 20 through a flexible third-degree polynomial $\varphi^{*}\left(\overrightarrow{W}\right)$estimated via OLS and obtaining the predicted $\widehat{\varphi}^{*}\left(\overrightarrow{W}\right)$.

We then estimate the system of demand and production functions replacing $\ln Q^{*}_{ft}$ with $\varphi^{*}\left(\overrightarrow{W}\right)$ in the production function. We use GMM methods and rely on the moment conditions presented in the main text for identification. Our estimates of production coefficients are initialized at the respective OLS estimates of the production function augmented with Λ_{ft}^{Q} and Λ_{ft}^{M} regressors (coefficients for Λ_{ft}^{Q} and Λ_{ft}^{M} also freely estimated by OLS). Our σ estimate is initialized through an IV estimation of demand function, where the instrument for Q is the residual from the OLS production function. The IV procedure follows the spirit of Foster, Haltiwanger and Syverson (2008), though only for initialization.

G Variance Decomposition

This appendix explains the structural form variance decomposition presented in Tables 3, 4, and 7 of the main text. We follow a two-stage procedure, similar to that in Hottman, Redding and Weinstein (2016).

G.1 Structural Decomposition

The structural decomposition for sales and sales growth is guided by:

$$\begin{aligned} R_{ft} &= d_{ft}^{\kappa_1} a_{ft}^{\kappa_2} pm_{ft}^{-\phi\kappa_2} w_{ft}^{-\beta\kappa_2} \mu_{ft}^{-\gamma\kappa_2} \left(\widehat{\chi}_t \chi_{ft}\right)^{1-\frac{1}{\sigma}} \\ \frac{R_{ft}}{R_{f0}} &= \left(\frac{d_{ft}}{d_{f0}}\right)^{\kappa_1} \left(\frac{a_{ft}}{a_{f0}}\right)^{\kappa_2} \left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\phi\kappa_2} \left(\frac{w_{ft}}{w_{f0}}\right)^{-\beta\kappa_2} \left(\frac{\mu_{ft}}{\mu_{f0}}\right)^{-\gamma\kappa_2} \left(\widehat{\chi}_t \chi_{ft}\right)^{1-\frac{1}{\sigma}} \end{aligned} \tag{21}$$

1. Guided by the above equation, we obtain $\ln \chi_{ft}$ as a residual from the following equation:

$$\begin{aligned} \ln \frac{R_{ft}}{R_{f0}} &= \beta_D \ln\left(\frac{d_{ft}}{d_{f0}}\right) + \beta_A \ln\left(\frac{a_{ft}}{a_{f0}}\right) + \beta_\mu \ln \frac{\mu_{ft}}{\mu_{f0}} \\ &\quad + \beta_M \ln\left(\frac{pm_{ft}}{pm_{f0}}\right) + \beta_w \ln\left(\frac{w_{ft}}{w_{f0}}\right) + \ln\left(\chi_{ft}\right)^{\left(1-\frac{1}{\sigma}\right)} \end{aligned} \tag{22}$$

where $\beta_D = \kappa_1$; $\beta_A = \kappa_2$; $\beta_\mu = -\gamma\kappa_2$; $\beta_M = -\phi\kappa_2$; $\beta_w = -\beta\kappa_2$; $\kappa_1 = \frac{1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}$; $\kappa_2 = \left(1-\frac{1}{\sigma}\right)\kappa_1$. We calculate these parameters using our estimates of factor elasticities in technology and the elasticity of substitution. Because we use these parameters that stem from the structure of the model, we label the residual as a "residual " wedge. The attributes d_{ft}, a_{ft}, pm_{ft} and w_{ft} correspond to the idiosyncratic components of demand, technology, and input price shocks, estimated as already described ($D_{ft} = D_t d_{ft}$ and so on).

2. We then estimate the following equations:

$$\begin{aligned}
\beta_D \ln\left(\frac{d_{ft}}{d_{f0}}\right) &= \rho_{0,D} + \rho_D \ln\frac{R_{ft}}{R_{f0}} + \nu_{ft,D} \\
\beta_A \ln\left(\frac{a_{ft}}{a_{f0}}\right) &= \rho_{0,A} + \rho_A \ln\frac{R_{ft}}{R_{f0}} + \nu_{ft,A} \\
\beta_\mu \ln\left(\frac{g\left(s_{ft}\right)}{g\left(s_{f0}\right)}\right) &= \rho_{0,\mu} + \rho_\mu \ln\frac{R_{ft}}{R_{f0}} + \nu_{ft,A} \\
\beta_M \ln\left(\frac{pm_{ft}}{pm_{f0}}\right) &= \rho_{0,M} + \rho_M \ln\frac{R_{ft}}{R_{f0}} + \nu_{ft,M} \\
\beta_w \ln\left(\frac{w_{ft}}{w_{f0}}\right) &= \rho_{0,w} + \rho_w \ln\frac{R_{ft}}{R_{f0}} + \nu_{ft,w} \\
\ln\widehat{\chi_{ft}} &= \rho_{0,\upsilon} + \rho_\upsilon \ln\frac{R_{ft}}{R_{f0}} + \nu_{ft,\upsilon}
\end{aligned} \tag{23}$$

We now prove that the contribution of each attribute to the variance of sales equals the ratio of its covariance with sales to the variance of sales multiplied by its structural parameter in equation 22. Also that, by the properties of OLS, the contribution of the different factors considered add up to 1. We conduct the proof for the two-covariance case for simplicity.

For any given log-linear equation (such as 22):

$$Y_f = \beta_1 X_{1f} + \beta_2 X_{2f} + \varepsilon_i \tag{24}$$

If one estimates by OLS The set of equations

$$\beta_1 X_{1f} = \gamma_{1,0} + \gamma_1 Y_f + \nu_{1i} \tag{25}$$

$$\beta_2 X_{2f} = \gamma_{1,0} + \gamma_2 Y_f + \nu_{2i} \tag{26}$$

and

$$\varepsilon_f = \gamma_{\varepsilon,0} + \gamma_\varepsilon Y_f + \nu_{\varepsilon f} \tag{27}$$

The estimated parameters for $j = \{1, 2\}$ are:

$$\begin{aligned}
\hat{\gamma_j} &= \frac{Cov(\beta_j X_{jf}, Y_f)}{Var(Y_f)} = \beta_j \frac{Cov(X_{jf}, Y_f)}{Var\left(Y_f\right)} \\
&= \beta_j Corr(X_{ji}, Y_f)\left(\frac{Var(X_{jf})}{Var\left(Y_f\right)}\right)^{\frac{1}{2}}
\end{aligned}$$

Since $\varepsilon_f = Y_f - (\beta_1 X_{1f} + \beta_2 X_{2f})$, $\hat{\gamma_\varepsilon}$ can be re-written as:

$$\begin{aligned}
\hat{\gamma_\varepsilon} &= \frac{Cov(Y_f - (\beta_1 X_{1f} + \beta_2 X_{2f}), Y_f)}{Var\left(Y_f\right)} \\
&= \frac{Var\left(Y_f\right) - \beta_1 Cov(X_{1f}, Y_f) - \beta_2 Cov(X_{2f}, Y_f)}{Var\left(Y_f\right)} = 1 - \hat{\gamma_1} - \hat{\gamma_2}
\end{aligned}$$

G.2 Decomposition by Ages

To conduct the decomposition by ages, we expand equations 22 and 23 to include interactions with the different age groups. Suppose there are two mutually exclusive groups: B and C. We redefine the equation 22 as:

$$Y_f = \beta_1 X_{1f} + \beta_2 X_{2f} + \varepsilon_i \tag{28}$$

$$\ln \frac{Q_{ft}}{Q_{f0}} = \beta_{1,C} X_{1f} d_{Cf} + \beta_{1,B} X_{1f} d_{Bf} \tag{29}$$

$$+\beta_{2,C} X_{2f} d_{Cf} + \beta_{2,B} X_{2f} d_{Bf} + \varepsilon_i \tag{30}$$

where $d_{Cf} = 1$ if f belongs to group C (say, an age), and everything else as defined previously.

The new decomposition equation for, say, X_1 will be given by:

$$\beta_{1,C} X_{1f} d_{Cf} + \beta_{1,B} X_{1f} d_{Bf} = \gamma_{C1} Y_f d_{Cf} + \gamma_{B1} Y_f d_{Bf} + \nu_{1f} \tag{31}$$

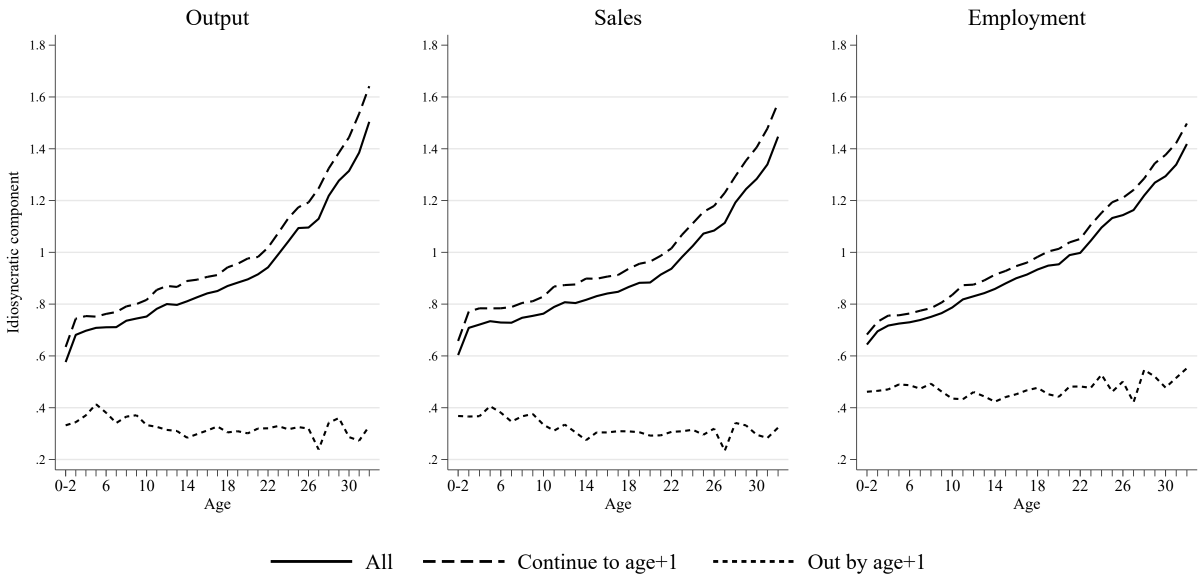
$$\varepsilon_f = \gamma_{C\varepsilon} Y_f d_{Cf} + \gamma_{B\varepsilon} Y_f d_{Bf} + \nu_{\varepsilon f} \tag{32}$$

Just as before $\hat{\gamma_{C1}} + \hat{\gamma_{C\varepsilon}} = \hat{\gamma_{B1}} + \hat{\gamma_{B\varepsilon}} = 1$.

H Selection

By construction, we focus on survivors and on growth life cycle growth from birth to age t of plants that have survived to age t. We contrast here survivors, defined as year t plants also present in $t+1$, with plants about to exit, which are year t plants not present in $t+1$. Figure H.1 illustrates that the size of exits-to-be departs significantly, downwards, from that of continuers. But, the differential size of plants that exit only affects marginally the overall average. That is, the average patterns described in the main text are mainly driven by plants present in t that continue to exist is $t+1$.

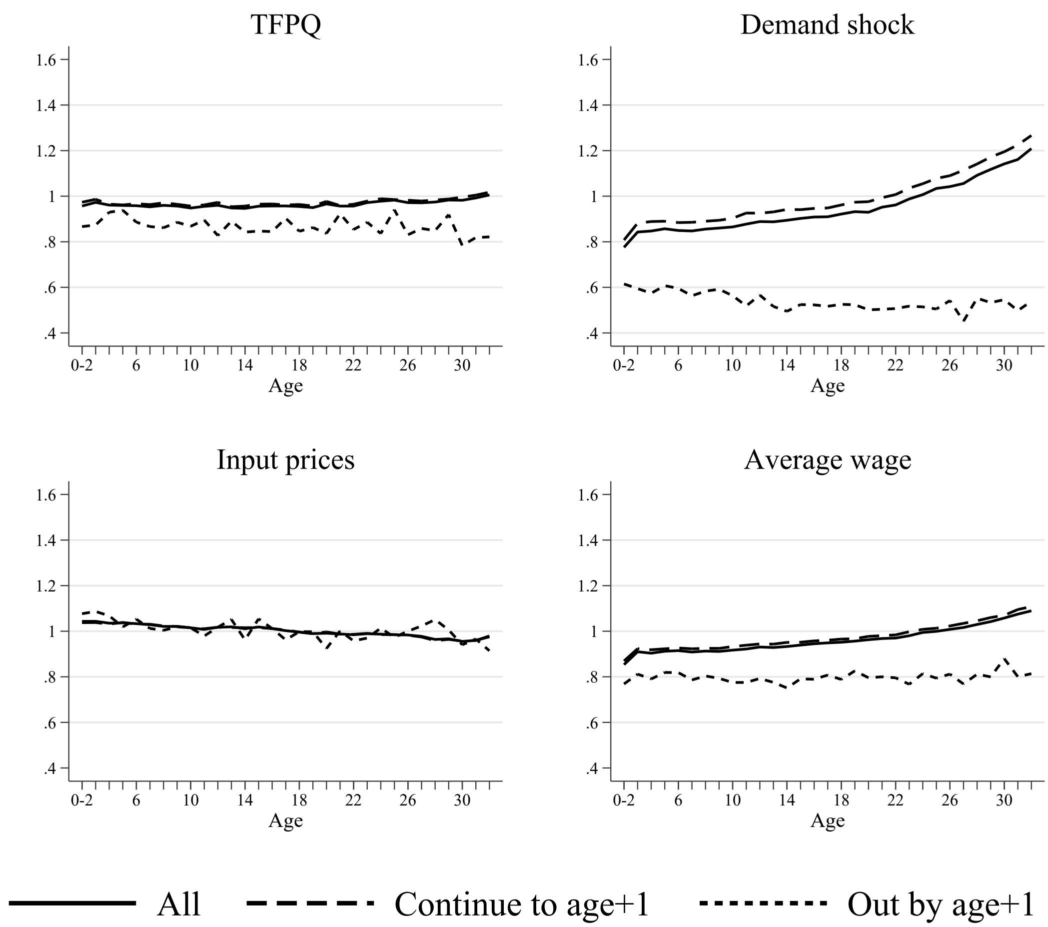
Figure H.1: Size Distribution of Exiters vs. Continuers



Note: Lines depict the average for each variable based only on idiosyncratic variation. Information from the sample of plant observations for which all measured attributes are observable.

Figure H.2 shows different attributes separately for continuers and exits-to-be. The most noteworthy difference is much poorer demand for plants about to exit compared to those that will continue, suggestive of demand side attributes being particularly important determinants of exit. $TFPQ$ is also lower for exits, although the $TFPQ$ premium of survivors is minor compared to their d_{ft} premium. Exits-to-be also pay lower wages, another sign of negatively-correlated wedges that allow low-productivity plants (such as exits-to-be) to expand beyond their efficient size, and are likely to survive beyond the efficient time.

Figure H.2: Attributes: Exiters vs. Continuers



Note: Lines depict the average for each attribute based only on idiosyncratic variation. Information from the sample of plant observations for which all measured attributes are observable.

Table H.1 further carries our variance decomposition of sales separately for these two groups of plants. Quality-adjusted productivity still plays an important role for exiters in explaining their size at the moment in which they are about to exit. Despite demand shocks being the dimension where most marked differences are observed between exits-to-be and continuers, especially for older ages (Figure H.2), $TFPQ$ tends to play a slightly more significant role in explaining the cross-sectional variance of size among exiters than among continuers.

Table H.1: Decomposition of Sales by Age: Exiters vs. Continuers

	Continuers				Exiters			
	Weighted avg. ages	Age 3	Age 10	Age 20	Weighted avg. ages	Age 3	Age 10	Age 20
TFPQ-HK	**1.116**	**1.151**	**1.119**	**1.108**	**1.146**	**1.126**	**1.139**	**1.076**
TFPQ	0.079	0.112	0.078	0.071	0.104	0.096	0.131	0.064
Demand	1.037	1.039	1.041	1.037	1.042	1.030	1.008	1.012
Composite (HK) wedge	**-0.116**	**-0.151**	**-0.119**	**-0.108**	**-0.146**	**-0.126**	**-0.139**	**-0.076**
Input prices	0.002	0.001	-0.001	0.005	-0.005	0.001	0.000	0.021
Wages	-0.061	-0.060	-0.057	-0.063	-0.070	-0.061	-0.055	-0.070
Markup	-0.015	-0.010	-0.011	-0.014	-0.007	-0.005	-0.004	-0.003
Residual wedge	-0.042	-0.083	-0.049	-0.036	-0.065	-0.062	-0.079	-0.024
Marginal cost HRW	**-0.022**	**-0.029**	**-0.029**	**-0.023**	**-0.036**	**-0.025**	**-0.003**	**-0.009**

Note: Weighted average across ages corresponds to the weighted average up to and including age 50. $TFPQ\_HK$ values correspond to the sum of the contributions of D and $TFPQ$; Composite (HK) wedge is the sum of the contributions of input prices, markups, and residual wedges; Marginal cost HRW is the sum of the contributions of $TFPQ$, input prices, and residual wedges. In this case we estimate a decomposition pooling across all sectors, noting that differences when doing so are small, because of the small number of plants that are exits-to-be.

I Cross-Sectional Variability with a Distortion Adjusted User Cost of Capital

While input prices and idiosyncratic markups can be measured directly from our data, the same is not true for the user cost of capital and factor-biased distortions. An indirect inference approach can be implemented, however. Revenue in our model can be written as

$$R_{ft} = d_{ft}^{\kappa_1} a_{ft}^{\kappa_2} pm_{ft}^{-\phi\kappa_2} w_{ft}^{-\beta\kappa_2} \mu_{ft}^{-\gamma\kappa_2} \left(\widehat{\chi}_t \chi_{ft}\right)^{1-\frac{1}{\sigma}} \tag{33}$$

where the residual wedge $\chi_{ft} = (1-\tau_{ft})^{\gamma\kappa_1} * \left(r_{ft}{\chi^{\mathrm{K}}}_{ft}\right)^{-\alpha\kappa_2}$ includes revenue distortions τ_{ft} and the idiosyncratic user cost of capital inclusive of distortions with respect to the prices of other inputs (factor-biased distortions). From the first-order conditions of capital and labor we obtain:

$$\frac{K_{ft}}{L_{ft}} = \frac{\alpha}{\beta} \frac{w_{ft}}{(r_{ft}{\chi^{\mathrm{K}}}_{ft})} \tag{34}$$

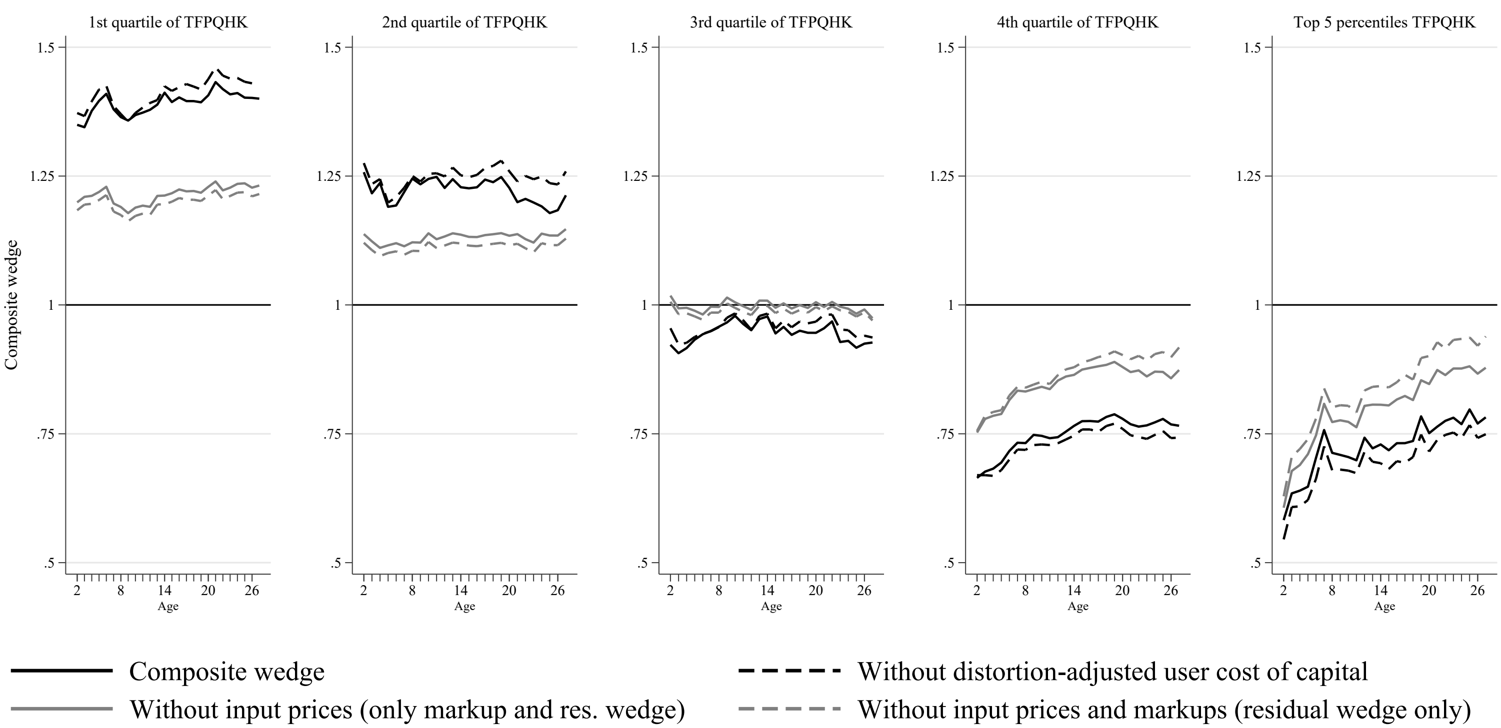
Solving for $\left(r_{ft}{\chi^{\mathrm{K}}}_{ft}\right)$ we can decompose χ_{ft} into its revenue and factor-specific components, respectively given by $(1-\tau_{ft})^{\gamma\kappa_1}$ and $\left(r_{ft}{\chi^{\mathrm{K}}}_{ft}\right)$. Table I.1 shows the result of the variance decomposition of sales separating the residual wedge into its revenue component and distortion-adjusted user cost of capital. We find that factor-biased component plays a minor role in the cross-sectional distribution of sales compared to the residual wedge. That is, most of the important role we find for residual wedges is driven by revenue wedges rather than factor-biased ones, including adjustment costs specific to investment in physical assets. Figure I.1 further shows the decomposition of the composite wedges dissecting the role of the (adjusted) user cost of capital. It shows that the role of factor-biased distortions is minor not only on average but across all ages and sections of the $TFPQ\_HK$ distribution. The adjusted user cost of capital is slightly more important for older plants, which seem to pay higher interests and thus become relatively undersized, and for plants at the top five percentiles of the composite productivity distribution, for which lower user cost of capital moderates the strongly negative composite wedges they face.

Table I.1: Variance Decomposition of Sales, Using Estimated Distortion-Adjusted User Cost of Capital

	Levels decomposition		Growth decomposition	
	Unweighted	Revenue Weighted	Unweighted	Revenue Weighted
TFPQ-HK	1.139	1.141	1.216	1.286
TFPQ	0.081	0.085	0.142	0.206
D (quality/appeal)	1.058	1.056	1.074	1.080
Composite (HK) wedge	-0.139	-0.141	-0.216	-0.286
Material prices	0.003	-0.003	-0.005	-0.003
Wages	-0.073	-0.073	-0.046	-0.043
Markup	-0.019	-0.077	-0.009	-0.031
Distortion-Adjusted User Cost of Capital	0.029	0.042	-0.020	0.005
Residual wedge	-0.079	-0.029	-0.136	-0.215
Marginal cost HRW	-0.039	0.021	-0.065	-0.050

Note: Each value corresponds to the weighted average across ages up to and including age 50. $TFPQ\_HK$ values correspond to the sum of the contributions of D and $TFPQ$; HK wedge is the sum of the contributions of input prices, markups, and residual wedges; Marginal cost HRW is the sum of the contributions of $TFPQ$, input prices, and residual wedges.

Figure I.1: Composite Wedges by Age: The Role of $TFPR$ and its Components With Distortion-Adjusted User Cost of Capital



Note: Lines depict average composite HK wedges and its components based only on idiosyncratic variation. Information from the sample of plant observations for which all measured attributed are observable.

J Hottman, Redding and Weinstein Framework Accounting Explicitly for Wedges

Our framework closely follows the modeling of the demand side in Hottman, Redding and Weinstein (2016). On the cost side, however, they model total costs rather than efficiency and input prices individually and do so at the product level rather than the firm level. They also abstract from wedges. Expanding HRW's framework to include wedges explicitly, and focusing on the case of uniproduct firms where their approach and ours are equivalent, the firm solves:

$$\underset{Q_{ft}}{Max} \left(1-\tau_{ft}\right) P_{ft} Q_{ft} - CT_{ft}\left(Q_{ft}\right)$$

where $CT_{ft}\left(Q_{ft}\right)$ is total cost as a function of output. Profit maximization leads to first orden condition $\left(P_{ft} + Q_{ft}\frac{dP_{ft}}{dQ_{ft}}\right) = \frac{\frac{\partial CT_{ft}}{\partial Q_{ft}}}{\left(1-\tau_{ft}\right)}$, so that at the optimum

$$\mu_{ft} = \frac{P_{ft}}{\frac{\partial CT_{ft}}{\partial Q_{ft}}\left(1-\tau_{ft}\right)^{-1}} \tag{35}$$

The associated optimal markup is given by (see appendix D):

$$\mu_{ft} = \frac{1}{1-\left(\frac{1}{\sigma}+\left(\frac{\sigma-1}{\sigma}\right)s_{ft}\right)} \tag{36}$$

Moreover, our demand structure is the same as in HRW. The implied demand function in the case of a uniproduct firm is:

$$Q_{ft} = d_{ft}^{\sigma}\left(\frac{P_{ft}}{P_t}\right)^{-\sigma}\frac{E_t}{P_t} \tag{37}$$

$$or$$

$$R_{ft} = d_{ft}^{\sigma}\left(\frac{P_{ft}}{P_t}\right)^{1-\sigma} E_t \tag{38}$$

$$\frac{P_{ft}}{P_t} = d_{ft}^{\frac{\sigma}{\sigma-1}} s_{ft}^{\frac{1}{1-\sigma}} \tag{39}$$

where $R_{ft} = P_{ft}Q_{ft}$ is firm sales and $s_{ft} = \frac{R_{ft}}{E_{ft}}$ is the firm's share in aggregate (sector) sales. Equation 37 is HRW's equation (5) for the uniproduct case (where $d_{ft} = \varphi_{ft}^{\frac{\sigma-1}{\sigma}}$ and φ_{ft} is the notation used in HRW). Equation 39 is obtained by direct manipulation of 38.

Replacing the optimal markup rule 35 into 38 HRW decompose firm sales into:

$$R_{ft} = d_{ft}^{\sigma}\frac{E_t}{P_t^{1-\sigma}}\left(\mu_{ft}\frac{\frac{\partial CT_{ft}}{\partial Q_{ft}}}{1-\tau_{ft}}\right)^{1-\sigma} \tag{40}$$

which is equivalent to HRW's equation (16). To see the equivalence, notice that in the uniproduct case $\frac{\partial CT_{fjt}}{\partial Q_{fjt}} = \frac{\partial CT_{ft}}{\partial Q_{ft}}$ (where j is a product and HRW have denoted by $\widetilde{\gamma}_{ft}$ the average marginal cost across products of a firm), and that $d_{ft} = \varphi_{ft}^{\frac{\sigma-1}{\sigma}}$. Firm sales variability can thus be decomposed into variation attributable to: 1) an aggregate component; 2) firm idiosyncratic demand d_{ft}; 3) firm markup; 4) a distortion-adjusted marginal cost $\frac{mc_{ft}}{(1-\tau_{ft})}$.

HRW's empirical procedure is as follows:

1) Estimate the demand function 37, in differences with respect to aggregates and over time, to obtain σ and decompose price (observable) into d_{ft} (not observable) and s_{ft} (observable).

2) Estimate the markup μ_{ft} based on observables, using 36.

3) With these components decompose the idiosyncratic variation of sales from equation 40 into the contributions of d_{ft}, μ_{ft} and the residual component: $\frac{\frac{\partial CT_{ft}}{\partial Q_{ft}}}{(1-\tau_{ft})}$. This is a distortion-adjusted marginal cost component, which HRW do not further decompose into its $\frac{\partial CT_{ft}}{\partial Q_{ft}}$ and $(1-\tau_{ft})$ components.

K Aggregate productivity

At the sector level (="aggregate") define aggregate TFP_t as in Hsieh and Klenow (2009) with our notation:

$$TFP_t = \frac{Q_t}{X_t} \tag{41}$$

With multiple inputs and X_{ft} as a Cobb Douglas aggregate of M_{ft},K_{ft},and L_{ft}, X_t will be a Cobb Douglas aggregate of $M_t = \sum_{I_t} M_{ft}$, $L_t = \sum_{I_t} L_{ft}$, and $K_t = \sum_{I_t} K_{ft}$. We assume that $Q_{ft} = A_{ft}X_{ft}^{\gamma}$ and define $TFPR_{ft} \equiv \frac{P_{ft}Q_{ft}}{X_{ft}}$. In this case

$$TFPR_{ft} \equiv \frac{P_{ft}Q_{ft}}{X_{ft}} = P_{ft}A_{ft}\frac{X_{ft}^{\gamma}}{X_{ft}} \tag{42}$$

It is also the case that, given that $P_{ft} = D_{ft}Q_{ft}^{-\frac{1}{\sigma}}$

$$\begin{aligned} TFPR_{ft} &\equiv \frac{P_{ft}Q_{ft}}{X_{ft}} = \frac{D_{ft}Q_{ft}^{1-\frac{1}{\sigma}}}{X_{ft}} \\ &= \frac{D_{ft}A_{ft}^{1-\frac{1}{\sigma}}}{X_{ft}^{1-\gamma\left(1-\frac{1}{\sigma}\right)}} \end{aligned} \tag{43}$$

Moreover, at the optimum

$$X_{ft} = \left(\frac{D_{ft}A_{ft}^{1-\frac{1}{\sigma}}\gamma}{C_{ft}\mu_{ft}\left(1-\tau_{ft}\right)^{-1}}\right)^{\frac{1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}} \tag{44}$$

Replacing (44) into (43) we obtain:

$$TFPR_{ft} = \frac{C_{ft}\mu_{ft}}{\gamma\left(1-\tau_{ft}\right)} \tag{45}$$

So that X_{ft} in (44) can be written:

$$X_{ft} = \left(\frac{D_{ft}A_{ft}^{1-\frac{1}{\sigma}}}{TFPR_{ft}}\right)^{\frac{1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}} \tag{46}$$

Replacing 46 into 42:

$$TFPR_{ft} = P_{ft}A_{ft}\left(\frac{D_{ft}A_{ft}^{1-\frac{1}{\sigma}}}{TFPR_{ft}}\right)^{\frac{\gamma-1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}}$$

so that

$$P_{ft} = \left(TFPR_{ft}A_{ft}^{\frac{-1}{\gamma}}D_{ft}^{\frac{(1-\gamma)\sigma}{\gamma}}\right)^{\frac{\gamma}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}$$

Also, working from the definition of TFP_t

$$\begin{aligned}
TFP_t &\equiv \frac{Q_t}{X_t} = \frac{P_tQ_t}{X_t}\frac{1}{P_t} = \frac{E_t}{X_t}\frac{1}{P_t} \\
&= \frac{E_t}{X_t}\left(\sum_{I_t} d_{ft}^{\sigma}P_{ft}^{1-\sigma}\right)^{\frac{1}{\sigma-1}} \\
&= \frac{E_t}{X_t}\left(\sum_{I_t} d_{ft}^{\sigma}\left(TFPR_{ft}A_{ft}^{\frac{-1}{\gamma}}D_{ft}^{\frac{(1-\gamma)\sigma}{\gamma}}\right)^{\frac{\gamma(1-\sigma)}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}\right)^{\frac{1}{\sigma-1}} \\
&= \left(\sum_{I_t}\left(\frac{d_{ft}^{\frac{\sigma}{(\sigma-1)}}D_{ft}^{\frac{(\gamma-1)}{\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}A_{ft}^{\frac{1}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}\overline{TFPR_t}}{TFPR_{ft}^{\frac{\gamma}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}\right)^{(\sigma-1)}\right)^{\frac{1}{\sigma-1}} \\
&= D_t^{-\frac{\sigma}{\sigma-1}}\left(\sum_{I_t}\left[\left(\frac{D_{ft}^{\frac{\sigma}{(\sigma-1)}}A_{ft}}{TFPR_{ft}^{\gamma}}\right)^{\frac{1}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}\overline{TFPR_t}\right]^{\sigma-1}\right)^{\frac{1}{\sigma-1}}
\end{aligned} \tag{47}$$

where we have defined $\overline{TFPR_t} = \frac{E_t}{X_t} = \frac{P_tQ_t}{X_t} = \left(\frac{P_tQ_t}{K_t}\right)^{\frac{\alpha}{\alpha+\beta+\phi}}\left(\frac{P_tQ_t}{L_t}\right)^{\frac{\beta}{\alpha+\beta+\phi}}\left(\frac{P_tQ_t}{M_t}\right)^{\frac{\phi}{\alpha+\beta+\phi}}$ and we have used $\frac{\sigma}{(\sigma-1)} + \frac{(\gamma-1)}{\left(1-\gamma\left(\frac{\sigma-1}{\sigma}\right)\right)} = \frac{1}{(\sigma-1)\left(1-\gamma\left(\frac{\sigma-1}{\sigma}\right)\right)}$. Additionally, defining $TFPQ\_HK_{ft} = D_{ft}^{\frac{\sigma}{(\sigma-1)}}A_{ft}$ one could write TFP_t as in equation (20) from the main text:

$$TFP_t = D_t^{-\frac{\sigma}{\sigma-1}} \left(\sum_{I_t} \left(\left(\frac{TFPQ\_HK_{ft}}{TFPR_{ft}^{\gamma}} \right)^{\frac{1}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}} \overline{TFPR_t} \right)^{\sigma-1} \right)^{\frac{1}{\sigma-1}} \tag{48}$$

L Components of Allocative Efficiency. The Role of Distortions

Starting from equation 48 and dividing it by its efficient level, allocative efficiency is given by (see main text)

$$AE_t = \left(\frac{1}{N_t} \sum_{I_t} \left[\left(\frac{\Delta_{ft}^{\frac{1}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}{\widetilde{\Delta}_t} \right) \left(\frac{tfpr_{ft}^{\frac{\gamma}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}{\overline{tfpr_t}} \right)^{-1} \right]^{\sigma-1} \right)^{\frac{1}{\sigma-1}} \tag{49}$$

$$= \left[cov\left(\left(\frac{\Delta_{ft}^{\frac{1}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}{\widetilde{\Delta}_t} \right)^{\sigma-1}, \left(\frac{tfpr_{ft}^{\frac{\gamma}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}{\overline{tfpr_{ft}}} \right)^{1-\sigma} \right) + E\left(\frac{tfpr_{ft}^{\frac{\gamma}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}{\overline{tfpr_{ft}}} \right)^{1-\sigma} \right]^{\frac{1}{\sigma-1}} \tag{50}$$

where $\Delta_{ft} = d_{ft}^{\frac{\sigma}{(\sigma-1)}} a_{ft} = \frac{TFPQ\_HK_{ft}}{A_t}$; $\widetilde{\Delta}_t = \left(\frac{1}{N_t} \sum_{I_t} \Delta_{ft}^{\frac{\sigma-1}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}} \right)^{\frac{1}{\sigma-1}}$; and $\overline{tfpr_t} = \left(\sum_{I_t} tfpr_{ft} \frac{X_{ft}}{X_t} \right)$.

The covariance term can be further decomposed into the product of the correlation coefficient and the product of standard deviations of the two terms. The following table shows how the estimated AE and the different counterfactual AE can be decomposed into these different components.

Table L.1: Allocative Efficiency and its Components. The Role of Distortions

	Sector type					
	All	Low revenue curvature parameter	Intermediate revenue curvature parameter	High revenue curvature parameter	Low markup dispersion	High markup dispersion
Panel A: Aggregate efficiency $\left(\frac{1}{N_t}\sum_{I_t}\left[\left(\frac{\Delta_{ft}^{\frac{1}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}{\tilde{\Delta}_t}\right)\left(\frac{tfpr_{ft}^{\frac{\gamma}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}{\widetilde{tfpr}_t}\right)^{-1}\right]^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$						
AE	0.624	0.727	0.651	0.401	0.671	0.405
Shutting down markups and wedges (only input price disp. remain)	0.679	0.807	0.715	0.392	0.732	0.430
Shutting down input prices and wedges (only markup disp. remain)	0.895	0.968	0.898	0.807	0.941	0.679
Shutting down input prices and markups (only wedges remain)	0.840	0.854	0.861	0.733	0.795	1.054
Shutting down wedges (only input price and markup disp. remain)	0.619	0.776	0.650	0.321	0.688	0.292
Shutting down markups (only input price disp. and wedges remain)	0.704	0.737	0.752	0.464	0.685	0.797
Shutting down input prices (only markup disp. and wedges remain)	0.762	0.855	0.761	0.667	0.787	0.640
Shutting down all (no TFPR dispersion)	1.000	1.000	1.000	1.000	1.000	1.000
Panel B: $cov\left(\left(\frac{\Delta_{ft}^{\frac{1}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}{\tilde{\Delta}_t}\right)^{\sigma-1}, \left(\frac{tfpr_{ft}^{\frac{\gamma}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}{\widetilde{tfpr}_{ft}}\right)^{1-\sigma}\right)$						
AE	-1.142	-0.139	-0.627	-4.427	-0.624	-3.596
Shutting down markups and wedges (only input price disp. remain)	-0.292	-0.086	-0.252	-0.678	-0.221	-0.626
Shutting down input prices and wedges (only markup disp. remain)	-0.138	-0.024	-0.118	-0.342	-0.073	-0.445
Shutting down input prices and markups (only wedges remain)	-0.253	-0.027	-0.136	-0.993	-0.247	-0.278
Shutting down wedges (only input price and markup disp. remains)	-0.357	-0.107	-0.330	-0.733	-0.272	-0.763
Shutting down markups (only input price disp. and wedges remains)	-0.743	-0.113	-0.394	-2.919	-0.520	-1.799
Shutting down input prices (only markup disp. and wedges remains)	-0.584	-0.055	-0.360	-2.105	-0.346	-1.708
Panel C: $corr\left(\left(\frac{\Delta_{ft}^{\frac{1}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}{\tilde{\Delta}_t}\right)^{\sigma-1}, \left(\frac{tfpr_{ft}^{\frac{\gamma}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}{\widetilde{tfpr}_{ft}}\right)^{1-\sigma}\right)$						
AE	-0.215	-0.293	-0.219	-0.119	-0.226	-0.161
Shutting down markups and wedges (only input price disp. remain)	-0.211	-0.251	-0.212	-0.170	-0.215	-0.194
Shutting down input prices and wedges (only markup disp. remain)	-0.590	-0.701	-0.596	-0.454	-0.590	-0.589
Shutting down input prices and markups (only wedges remain)	-0.023	-0.106	0.000	-0.040	-0.066	0.176
Shutting down wedges (only input price and markup disp. remain)	-0.256	-0.300	-0.262	-0.180	-0.259	-0.240
Shutting down markups (only input price disp. and wedges remain)	-0.147	-0.251	-0.134	-0.103	-0.186	-0.035
Shutting down input prices (only markup disp. and wedges remain)	-0.144	-0.186	-0.147	-0.091	-0.146	-0.134
Panel D: $sd\left(\frac{tfpr_{ft}^{\frac{\gamma}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}{\widetilde{tfpr}_{ft}}\right)^{1-\sigma}$						
AE	5.150	0.178	2.472	21.965	2.447	17.945
Shutting down markups and wedges (only input price disp. remain)	0.365	0.128	0.333	0.748	0.307	0.640
Shutting down input prices and wedges (only markup disp. remain)	0.053	0.012	0.046	0.126	0.035	0.138
Shutting down input prices and markups (only wedges remain)	2.543	0.109	2.091	7.002	1.888	5.645
Shutting down wedges (only input price and markup disp. remain)	0.378	0.132	0.347	0.762	0.317	0.666
Shutting down markups (only input price disp. and wedges remain)	3.872	0.172	2.277	14.637	2.270	11.454
Shutting down input prices (only markup disp. and wedges remain)	3.647	0.112	2.294	13.181	2.067	11.127
Panel E: $E\left(\frac{tfpr_{ft}^{\frac{\gamma}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}}{\widetilde{tfpr}_{ft}}\right)^{1-\sigma}$						
AE	1.784	1.031	1.304	4.660	1.336	3.907
Shutting down markups and wedges (only input price disp. remain)	0.978	1.011	0.988	0.903	0.982	0.961
Shutting down input prices and wedges (only markup disp. remain)	1.030	1.011	1.029	1.054	1.021	1.073
Shutting down input prices and markups (only wedges remain)	1.087	0.969	0.993	1.621	1.064	1.198
Shutting down wedges (only input price and markup disp. remain)	0.990	1.019	1.005	0.899	0.994	0.975
Shutting down markups (only input price disp. and wedges remain)	1.449	1.008	1.149	3.215	1.244	2.421
Shutting down input prices (only markup disp. and wedges remain)	1.354	0.998	1.143	2.641	1.156	2.289
Number of sectors	23	5	16	2	19	4
Range of parameter		$[0.18, 0.31]$	$[0.37, 0.6]$	$[0.65, 0.69]$	< 0.1	> 0.1

M Alternative estimation methods

This appendix describes in detail each of the alternative methods to estimate the production and demand functions, the results of which are presented in section 7 of the main text.

Cost Shares, CS We assume constant returns to scale and estimate production elasticities for labor, material inputs, and capital as the respective cost shares (averaged across plants within each sector). We use direct observations on the wage bill and the cost of material inputs for the first two, and assume a rental rate of $r = 12\%$ (as in De Loecker, Eeckhout and Unger, 2020) to obtain the cost of capital as $r * K$. Since we are tying our hands to use only R_{ft} data (rather than production or price data), we follow HK to impose $\sigma = 3$ in this cost share method.

Proxy methods We estimate the production function specified in equation (20) in the appendix, but using revenue as the dependent variable and materials costs rather than our internally-deflated materials. We use our original control function ((19) in the appendix), which is derived from the First Order Condition for materials in the plant's problem. However, because in these alternative methods we tie our hands to use revenue and materials costs rather than quantities–in line with most of the literature–, compared to Appendix' equation (19) the control function excludes $\ln \frac{\overline{P^*_{ft}P_{fB}}}{PM^*_{ft}PM_{fB}}$, Λ^Q_{ft}, and Λ^M_{ft}. We also use the cost of material inputs rather than their quantity.

The moment conditions used for identification vary across the different versions of proxy estimations that we use, as do the revenue function specifications. In particular:

- ACF, following (Ackerberg, Caves and Frazer, 2015). The following moments are used for identification:

$$E\begin{bmatrix} \ln(Pm * M)_{ft-1} \times \xi^A_{ft} \\ \ln L_{ft} \times \xi^A_{ft} \\ \ln K_{ft} \times \xi^A_{ft} \\ \ln A_{ft} \end{bmatrix} = 0 \tag{51}$$

  As in our Cost Share case, we impose $\sigma = 3$.

- DEU, following (De Loecker, Eeckhout and Unger, 2020): We proceed as in the ACF version described above, but the revenue function is written in the following way, that recognizes the absence of proper plant deflators and thus controls for market shares (see (De Loecker, Eeckhout and Unger, 2020)):

$$\ln R_{ft} = \alpha \ln K_{ft} + \beta \ln L_{ft} + \phi \ln(Pm_{ft} * M_{ft}) + \delta \ln s_{ft} + \ln A_{ft} \tag{52}$$

  The moment conditions are:

$$E\begin{bmatrix} \ln(Pm * M)_{ft-1} \times \xi^A_{ft} \\ \ln L_{ft} \times \xi^A_{ft} \\ \ln K_{ft} \times \xi^A_{ft} \\ \ln s_{ft} \times \xi^A_{ft} \\ \ln A_{ft} \end{bmatrix} = 0 \tag{53}$$

  We impose $\sigma = 3$.

- KG, Blackwood et al. (2021) propose a way to use insights from Klette and Griliches (1996) to address the biases generated by the use of revenue as a proxy for production in the production function estimation. The method also yields an estimate for σ.

In particular, using $P_{ft} = D_t d_{ft} Q_{ft}^{-\frac{1}{\sigma}}$ and its implication that $P_t = D_t Q_t^{-\frac{1}{\sigma}}$ we have that $\frac{P_{ft}}{P_t} = Q_t^{\frac{1}{\sigma}} Q_{ft}^{-\frac{1}{\sigma}} d_{ft}$. Thus, R_{ft} can be written:

$$R_{ft} = P_{ft} Q_{ft} = P_t Q_t^{\frac{1}{\sigma}} Q_{ft}^{1-\frac{1}{\sigma}} d_{ft} = P_t Q_t^{\frac{1}{\sigma}} \left(A_{ft} X_{ft}^{\gamma}\right)^{1-\frac{1}{\sigma}} d_{ft} \tag{54}$$

Based on this implication, Blackwood et al. (2021) estimate the following version of the revenue function:

$$\ln R_{ft} = \overline{\alpha} \ln K_{ft} + \overline{\beta} \ln L_{ft} + \overline{\phi} \ln(Pm_{ft} * M_{ft}) + \frac{1}{\sigma} \ln E_t + \left(\left(1 - \frac{1}{\sigma}\right)(\ln A_{ft} + \ln P_t) + \ln d_{ft}\right) \tag{55}$$

where $\overline{\alpha} = \alpha\left(1 - \frac{1}{\sigma}\right)$, $\overline{\beta} = \beta\left(1 - \frac{1}{\sigma}\right)$, $\overline{\phi} = \phi\left(1 - \frac{1}{\sigma}\right)$, and $E_t = Q_t P_t$. The parameter that accompanies E_t allows us to estimate $(\left(1 - \frac{1}{\sigma}\right)$ so that we can obtain the production elasticities by adjusting the estimated revenue elasticities correspondingly.

Blackwood et al. (2021) treat Q_t in the moment conditions in a way analogous to the way they treat L_{ft} and K_{ft}. Thus, the moment conditions we use are

$$E \begin{bmatrix} \ln(Pm * M)_{ft-1} \times \xi_{ft}^A \\ \ln L_{ft} \times \xi_{ft}^A \\ \ln K_{ft} \times \xi_{ft}^A \\ \ln E_t \times \xi_{ft}^A \\ \ln A_{ft} \end{bmatrix} = 0 \tag{56}$$

We also estimate a version where we use the lagged value of E_t in the moment conditions (i.e. we impose orthogonality between E_{t-1} and ξ_{ft}^A.

Uniproduct: De Loecker et al. (2016) suggest the use of the sample of uniproduct plants as an alternative for the need to aggregate across products in multi-product units. We also estimate a version of our baseline estimation restricting the sample to uniproduct establishments, for which output Q_{ft} corresponds to physical quantity of the (homogeneus) product. We define as uniproduct an establishment that produces a single product which is the same for all years. The estimation and moment conditions are identical to the baseline, and thus provides estimations of production elasticities and σ, but note that in this case $\ln \Lambda_{ft}^Q = 0$.[4]

OLS demand estimation: To assess the importance of having access to production data to form our instrument for demand, we also carry an OLS estimation of demand function (equation 26 of the main text) to estimate σ. Such estimation takes advantage of the information on P_{ft} and Q_{ft} but ignores the information on input use that is taken advantage of in our baseline joint estimation to identify σ.

[4]Since the uniproduct plant does have multiple inputs, the estimation is still able to estimate σ_w, but relying on information from this limited source of variation

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